Partial Differential Equations and Non-convex Geometry

Partial differential equations (PDEs) are equations with infinite sets of variables that are used to model natural phenomena like sound, fluid dynamics, and elasticity, and are often used to find some minimized value for a system (e.g., what is the optimal shape for a soap bubble suspended within a wire ring). Mathematicians are constantly finding new ways to improve the accuracy and predictive ability of PDE models because they can describe many phenomena and structures. Some of the most interesting structures are the most complicated, and these fall into a category called non-convex geometries. Non-convex geometries are modeled using non-linear PDEs. In contrast to linear PDEs, which are well-behaved functions that are relatively straightforward to understand, nonlinear PDEs are complex and difficult, and while there are a range of basic mathematical tools that can be applied to them, there are no standard strategies that can be used, so each one must be evaluated and developed as its own separate puzzle.

Topology

Topology is a field that uses mathematics to describe the features of geometric objects. Dr. X is a pure mathematician, meaning he studies questions that are abstract in nature. The structures studied in topology are conceptual, but insight gained from the research performed in Dr. X's program have tangible applications to subjects like data management and biology. In general, the features that are described in topology are the type that remain consistent even if the object undergoes a continuous deformation. A continuous deformation is a kind of shape change that results from an object bending or stretching; this is opposed to a discontinuous deformation like cutting. Topology covers features, like connectedness and compactness, that are preserved as an object changes shape in this specific way.

Numerical Analysis

Numerical analysis is a division of math that solves difficult problems by approximation. The field tries to make approximations that are as accurate as possible to obtain the best solutions. Much of Dr. X's research is concerned with minimizing the amount of error in the models he creates by improving the way approximations are made. Specifically, Dr. X performs discretizations, which is a mathematical way of taking a continuous function (something that describes a feature that has continuity, like a smooth surface) and breaking it into discrete points that can be used for calculations. One challenge in this field is determining the most accurate ways to generate these points, and Dr. X works to develop and improve discretization algorithms to make them work better for a variety of applications.